WAVES

Wave Motion:- It is the phenomenon of transference of a disturbance from one point to the other with the exchange of energy and momentum, but without the transfer of matter.

Qn. Distinguish between transverse and longitudinal waves.

Transverse waves	**Longitudinal waves**
1. The waves are propagated in a direction right angle to the vibration of particles.	1. The waves are propagated in a direction parallel to the vibration of particles.
2. They are propagated as crests and troughs.	2. They are propagated as compressions and rarefactions.
3. They can travel through solids and surface of liquids.	3. They can travel through solids, liquids and gases.
4. They can be polarized.	4. They cannot be polarized.
5. Examples: water waves, waves formed in a stretched string, waves due to the up and down motion of slinky etc.	5. Examples: sound waves, waves due to the horizontal motion of slinky etc.

Characteristics of waves:

i. **Wavelength (λ)** It is the distance travelled by a single wave. It is measured as the distance between two consecutive crests or troughs, in the case of transverse waves and the distance between two consecutive compressions or rarefactions in the case of longitudinal waves. Unit: m

ii. **Frequency (ν):** It is the number of waves traveling per sec. Unit: Hertz

iii. **Time period (T):** It is the time taken by a single wave to travel. Unit: s

iv. **Phase (θ):** It is the argument of sine or cosine function in the equation that represents a wave. Unit: radians

v. **Amplitude (A):** It is the maximum displacement of a particle from its mean position. Unit: m

vi. **Wave velocity (v):** It is the distance traveled by waves in unit time. Unit: m/s

Relation between velocity, wavelength and frequency

Distance traveled by a single wave = Wavelength (λ)
Time taken by a wave = Time period (T)
So, wave velocity (v) = Distance / time
= λ / T (1/T = ν)

$$\mathbf{v = \nu\lambda}$$

Speed of waves

Transverse Waves in a stretched string

$v = \sqrt{T/\mu}$ Where T – tension in the string and μ - the linear density (mass per unit length of the string)

Speed of longitudinal waves

i. Through solids:
$\mathbf{v = \sqrt{Y/\rho}}$ where Y – Young's modulus and ρ - the density of the medium

ii. Though liquids and gases:
$\mathbf{v = \sqrt{B/\rho}}$ where B – Bulk modulus and ρ - the density of the medium

Speed of sound in air

Newton's formula

According to Newton, the propagation of sound in air is isothermal process.
So $v = \sqrt{B_i / \rho}$ --------(1)
As the process is isothermal, PV = a constant
Differentiating we get, $\Delta PV + \Delta VP = 0$

$\therefore P = -\Delta PV/\Delta V = B_i$

$$\mathbf{v = \sqrt{P/\rho}} \quad \text{-------------}\mathbf{(2)}$$

Laplace's correction

In equation (2), if we substitute the values of $P = 1.01 \times 10^5$ Pa and $\rho = 1.29$ kg / m^3 at S.T.P, we get velocity of sound as 280 m/s. But the experimental value obtained is nearly 332m/s under these conditions. This discrepancy is removed by Laplace, as given below. According to Laplace, the propagation of sound in air is adiabatic.

Hence, $v = \sqrt{B_a / \rho}$

As the process is adiabaic, PV^γ = a constant
Differentiating we get, $\Delta PV^\gamma + \gamma V^{\gamma-1} \Delta VP = 0$

$\therefore \gamma P = -\Delta PV^\gamma/\Delta V V^{\gamma-1} = -\Delta PV/\Delta V = B_a$

$$\boxed{\mathbf{v = \sqrt{\gamma P/\rho}}} \quad \text{-------------}\mathbf{(3)}$$

<u>*Factors affecting velocity of sound in air.*</u>

i. **Pressure**: When pressure increases, density also increases. Thus the ratio P/ρ is a constant. So the speed of sound is independent of pressure changes (refer eqn. 3 above), at constant temperature.

ii. **Temperature**: Density $\rho = M/V$

Hence eqn.3 becomes, $v = \sqrt{\gamma PV/M}$

$v = \sqrt{\gamma RT/M}$

$v \propto \sqrt{T}$

Thus the speed of sound is directly proportional to the square root of absolute temperature of air.

iii. **Density:** From the eqn.3, $v \propto 1/\sqrt{\rho}$

i.e., the speed of sound in a medium is inversely proportional to its density.

iv. **Humidity**: When humidity of atmosphere is high, the density of air is low. Thus speed of sound increases. (refer eqn. 3). Hence, the speed of sound in air increases with humidity.

v. **Wind:** The speed sound in air changes according to the direction of wind.

Progressive or Harmonic wave:

An unobstructed and un-damped wave proceeding with a given velocity is called a progressive wave.

Equation: **$y = A \sin(\omega t - kx + \phi)$,** if wave travels along the +X direction and
$y = A \sin(\omega t + kx + \phi)$, if wave travels along the - X direction.
Where y - particle displacement, x - the displacement of the wave, A – the amplitude of vibration of particles of the medium, $\omega = 2\pi\nu$ - the angular frequency of the wave, $k = 2\pi / \lambda$ - the wave number and ϕ - the initial phase.

<u>*Characteristics of progressive waves:*</u>

i. They are unobstructed and un-damped in the medium.
ii. Each particle in the medium is in S.H.M with the same amplitude and frequency.
iii. Particles separated by an integral multiple of λ have the same displacement, velocity and acceleration at any instant.
iv. Energy is transferred from particle to particle.

Reflection of waves:

When a progressive wave represented by **$y_i = A \sin(\omega t - kx)$** is struck on a rigid boundary, it gets reflected and the reflected wave is given by,

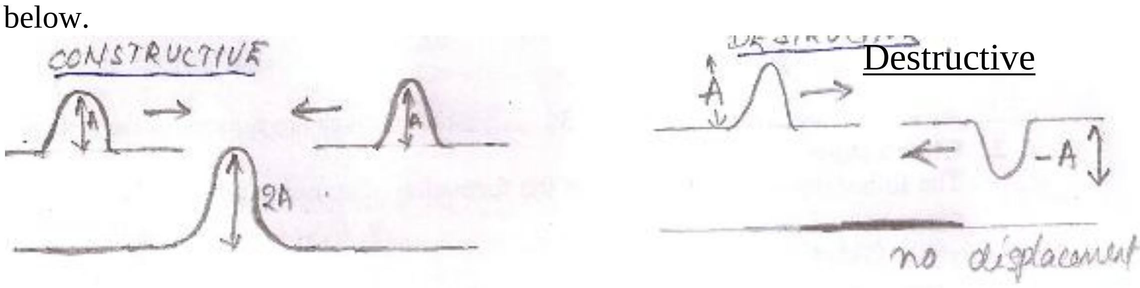
$$y_r = -A \sin(\omega t + kx)$$

Principle of superposition of waves

It states that when two or more waves overlap each other, the resultant displacement of a particle in the medium is the vector sum of individual displacements of the particle due to the different waves.

$$y = y_1 + y_2 + y_3 + \ldots\ldots$$

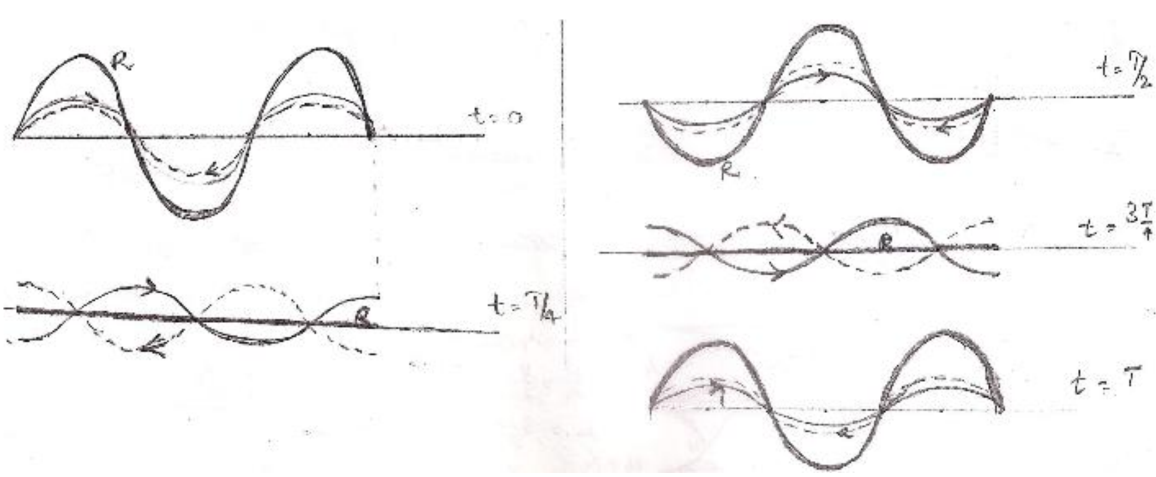
Constructive and destructive interferences between two identical wave pulses are shown below.



Standing waves or stationary waves

When two progressive waves of equal frequency and amplitude traveling in opposite directions along a straight line are superposed on one another, a standing (stationary) wave is formed.

<u>*Graphical treatment of the formation of standing waves.*</u>



Equation for a standing wave

$y = 2A \sin kx . \cos\omega t$

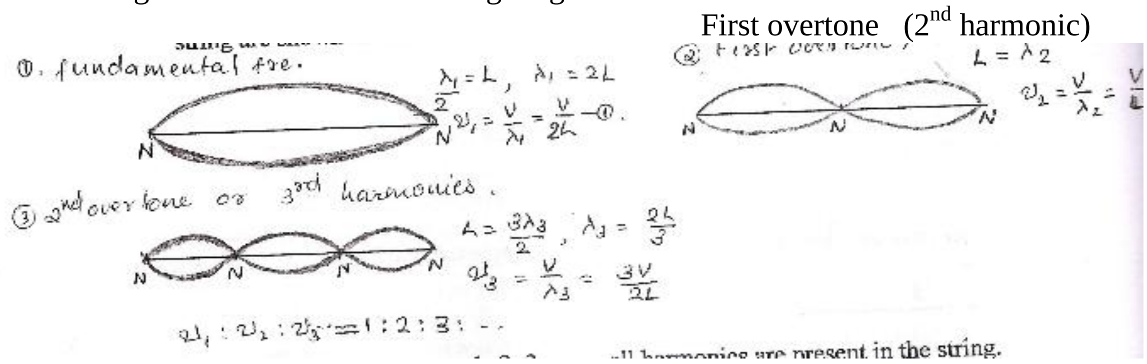
Characteristics of standing waves

(a). They are not progressive i.e., the crests and troughs do not travel forward.
(b). There is not transfer of energy between particles.
(c). Particles at different points vibrate with different amplitudes.
(d). The amplitude changes from zero to maximum. The points where the amplitude is zero are called **nodes** and the points where the amplitude is maximum are called **antinodes.**
(e). Every particle, except at nodes, executes S.H.M with the same period.
(f). The distance between two consecutive nodes or antinodes is $\lambda/2$.

Examples of standing waves.

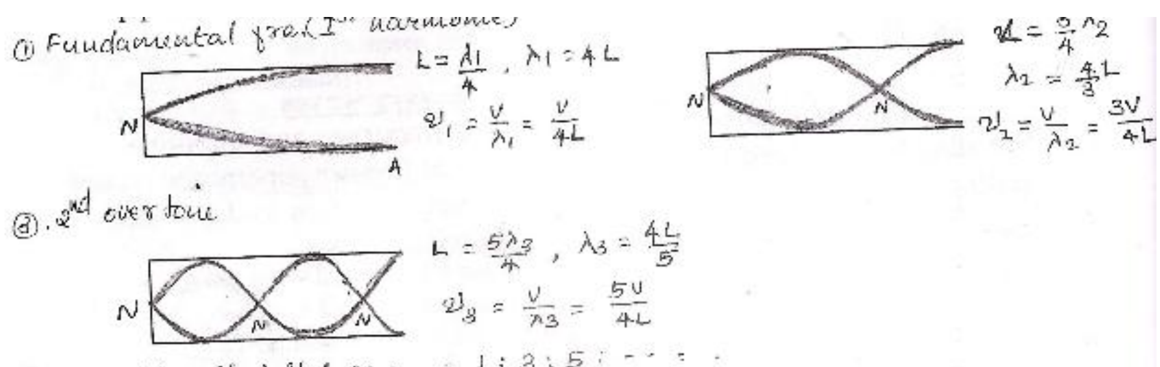
1. **Stretched string:**
   The different frequencies that are formed in the string are called **harmonics**. The lowest frequency that can be formed in the string is its fundamental frequency and its multiples are called **overtones**. The standing waves formed in a stretched string are shown in the following diagrams.

First overtone (2nd harmonic)



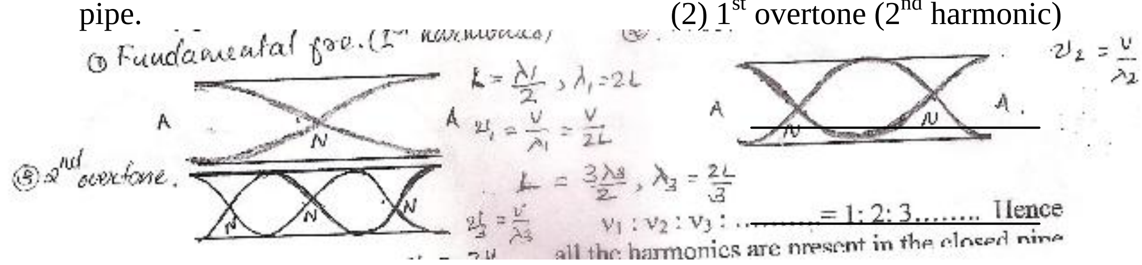
   As $\nu_1 : \nu_2 : \nu_3$:………….= 1: 2: 3:……, all harmonics are present in the string.

2. **Closed pipe:**
   The following diagrams represent the formation of standing waves in a closed pipe.
   (i) Fundamental freq. \(1st harmonic) (ii) 1st overtone (2nd harmonic)



   $\nu_1 : \nu_2 : \nu_3$: ……….= 1: 3: 5…….. Hence only odd harmonics are present in the closed pipe.

3. **Open pipe:**
   The following diagrams represent the formation of standing waves in an open pipe. (2) 1st overtone (2nd harmonic)



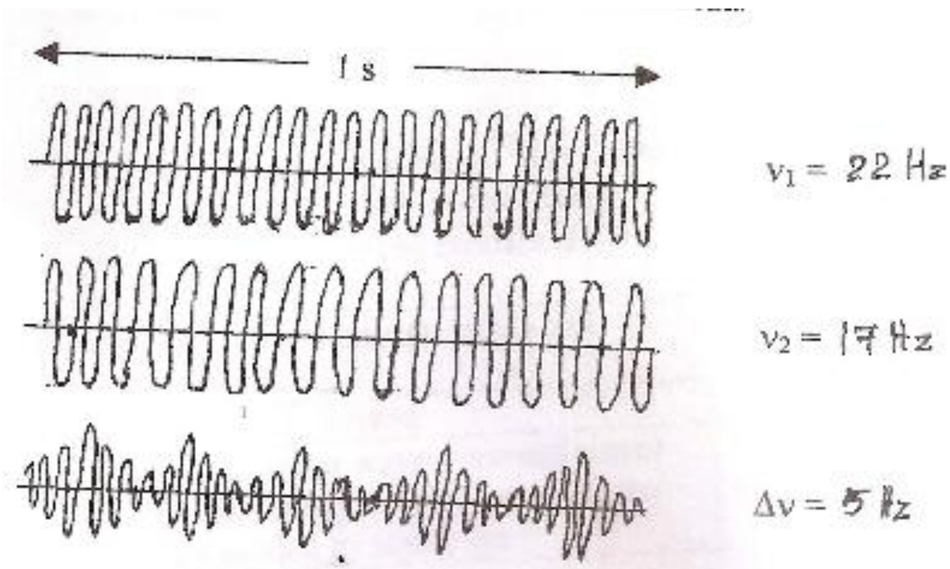
$\nu_1 : \nu_2 : \nu_3$: ……….= 1: 2: 3

Hence, all the harmonics are present in the closed pipe.

BEATS

It is the phenomenon of periodic variation in the intensity of the wave resulting from the super position of two waves of slightly different frequencies.

Graphical treatment to show the formation of beats



Analytical treatment

(Qn. Show that beat frequency is the difference between the parent frequencies)

Consider two waves of the same amplitude and slightly different frequencies ν_1 and ν_2 given by,

$y_1 = A \sin 2\pi\nu_1 t$ and $y_2 = A \sin 2\pi\nu_2 t$

When they overlap each other, according to the principle of superposition,

Resultant displacement $y = y_1 + y_2$

$y = A \sin 2\pi\nu_1 t + A \sin 2\pi\nu_2 t$

$= 2A [\sin 2\pi(\nu_1 + \nu_2)t /2 . \cos 2\pi(\nu_1 - \nu_2)t /2]$

$y = 2A \cos \pi(\nu_1 - \nu_2)t . \sin \pi(\nu_1 + \nu_2)t$

Thus the resultant amplitude is given by,

$R = 2A \cos \pi(\nu_1 - \nu_2)t$ ----------------(1)

The above equation shows that the amplitude of the resultant wave so formed varies periodically. As intensity is directly proportional to the square of amplitude ($I \alpha R^2$), there is periodic variation of intensity of the wave.

Intensity is maximum, when R is maximum. i.e., $\cos \pi(\nu_1 - \nu_2)t = \pm 1$.

i.e., $\pi(\nu_1 - \nu_2)t = 0, \pi, 2\pi, \ldots\ldots..$

So, intensity is max. at the instants $t = 0, 1/ (\nu_1 - \nu_2), 2 / (\nu_1 - \nu_2)\ldots\ldots\ldots$

$\therefore$ the time interval between two maxima $= 1/ (\nu_1 - \nu_2)$

i.e., Beat frequency [the number of beats per second] $(\Delta\nu) = (\nu_1 - \nu_2)$

Intensity is minimum, when R is minimum. i.e., $\cos \pi(\nu_1 - \nu_2)t = 0$.

i.e., $\pi(\nu_1 - \nu_2)t = \pi/2, 3/2\, \pi, 5/2\, \pi \ldots\ldots..$

So, intensity is min. at the instants $t = 1/ 2(\nu_1 - \nu_2), 3 / 2(\nu_1 - \nu_2), 5 / 2(\nu_1 - \nu_2), \ldots\ldots\ldots$

$\therefore$ the time interval between two minima $= 1/ (\nu_1 - \nu_2)$

Thus again, Beat frequency [the number of beats per second] $(\Delta\nu) = (\nu_1 - \nu_2)$

This shows that beat frequency is the difference between parent frequencies.

DOPPLER EFFECT

It is the phenomenon of apparent change in the frequency as heard by a listener, whenever there is a relative motion between the source and the listener.
Expression for apparent frequency

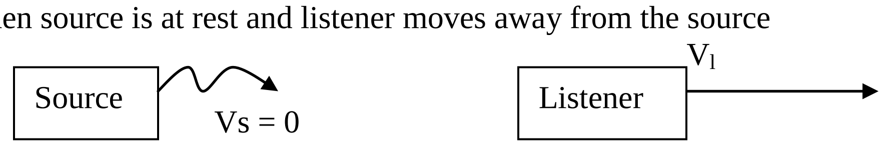
Consider a source moving with a velocity V_s, emitting sound of frequency ν and the listener also moving in the same direction with a velocity V_l. Let V be the speed of sound.

Relative velocity of sound with respect to the source = $V - V_s$
The apparent wavelength (λ') $= (V - V_s) / \nu$
Relative velocity of sound with respect to the listener = $V - V_l$
$\therefore$ The apparent frequency heard by the listener $= (V - V_l) / \lambda'$

i.e., $\underline{\boldsymbol{\nu' = \{(V - V_l) / (V - V_s)\}\nu}}$

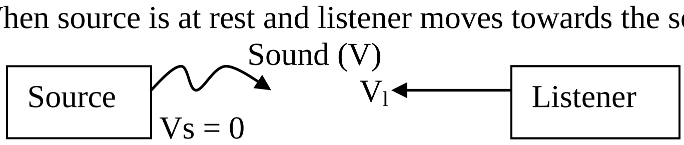
Special cases:

1. When source is at rest and listener moves away from the source



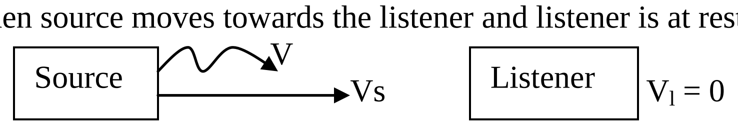
$\boldsymbol{\nu' = \{(V - V_l) / V\}\nu}$

2. When source is at rest and listener moves towards the source



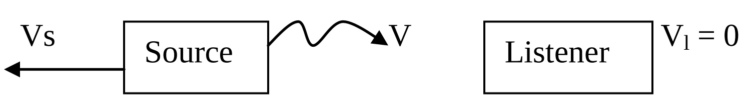
$\boldsymbol{\nu' = \{(V + V_l) / V\}\nu}$

3. When source moves towards the listener and listener is at rest.



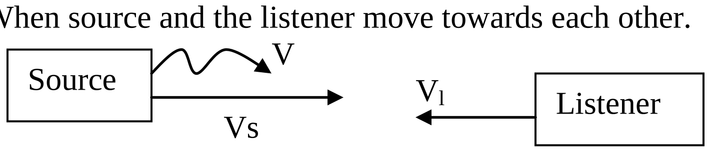
$\boldsymbol{\nu' = \{V / (V - V_s)\}\nu}$

4. When source moves away from the listener and listener is at rest.



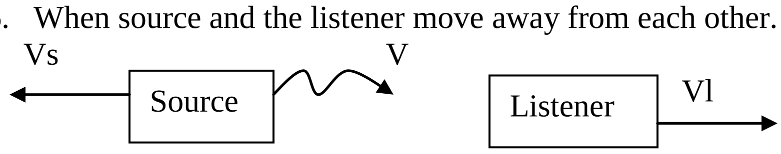
$\boldsymbol{\nu' = \{V / (V + V_s)\}\nu}$

5. When source and the listener move towards each other.



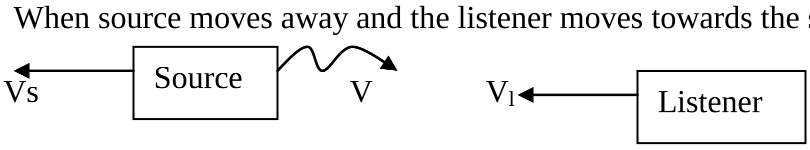
$\boldsymbol{\nu' = \{(V + V_l) / (V - V_s)\}\nu}$

6. When source and the listener move away from each other.



$\boldsymbol{\nu' = \{(V - V_l) / (V + V_s)\}\nu}$

7. When source moves away and the listener moves towards the source.



$\boldsymbol{\nu' = \{(V + V_l) / (V + V_s)\}\nu}$