Alternating current (A.C) is the current that changes in magnitude direction continuously with respect to time.

It can be represented as,

\[ V = V_m \sin \omega t \]
\[ I = I_m \sin \omega t \]
\[ T = \sin \omega t \]

The currents and voltages in a.c circuits can be expressed by the following terms:-

(i) **Instantaneous (I or V)**: It is the current or voltage that in the circuit at any instant.

(ii) **Peak (I_m or V_m)**: It is the maximum available voltage or current in the circuit.

(iii) **Average (I_av or V_av)**: It is the arithmetic mean of the instantaneous values of voltages or currents in the circuit.

(iv) **Room Mean Square (R.M.S) or Effective**: It is the square root of the average of the squares of the instantaneous values of the voltages or currents in the circuit.

\[ \text{R.M.S value} = \sqrt{\frac{1}{T} \int_0^T I^2 \, dt} \]

**Relation between Peak and R.M.S values**

\[ I_{rms}^2 = \frac{1}{T} \int_0^T I^2 \, dt = \frac{1}{T} \int_0^T I_m^2 \sin^2 \omega t \, dt \]

where \( \omega = \frac{2\pi}{T} \)

\[ I_{rms}^2 = \frac{I_m^2}{2} \int_0^T 1 - \cos 2\omega t \, dt \]

But \( \int_0^T \cos 2\omega t \, dt = \frac{\sin 2\omega t}{2\omega} \bigg|_0^T = 0 \)

Thus equation (1) becomes,

\[ I_{rms}^2 = \frac{I_m^2}{2} \int_0^T 1 - \cos 2\omega t \, dt \]

\[ I_{rms} = \frac{I_m}{\sqrt{2}} \]

**Note**: the measurement of ac is done by the comparison of the heating effect produced by ac with that by dc. Hence ac measuring instruments are also called ‘hot-wire instruments’.

Show that the average current in the complete cycle of a.c is zero.

\[ I_m = \int_0^T I \, dt = \int_0^T \frac{T_0 \text{Inst.} \, dt}{T} = \frac{I_m}{T} \int_0^T \text{Sin.} \, dt = \frac{I_m}{T} \left[ -\frac{\cos \omega t}{\omega} \right]_0^T \]

\[ = - \frac{I_m}{\omega} \left[ \cos \frac{2\pi}{T} \right]_0^T = - \frac{I_m}{\omega} \left[ \cos 2\pi - \cos 0 \right] = 0 \quad \text{(Hence proved)} \]

**Note**: But in half a cycle, \( I_m = \int_0^{T/2} I \, dt = \frac{1}{2} \int_0^T \text{Sin.} \, dt = 2 \frac{I_m}{T} \int_0^{T/2} \text{Sin.} \, dt = 2 \frac{I_m}{T} \left[ -\frac{\cos \omega t}{\omega} \right]_0^{T/2} \)

\[ = - \frac{2I_m}{\omega} \left[ \cos \frac{\pi}{T} \right]_0^{T/2} = - \frac{2I_m}{\omega} \left[ \cos \pi - \cos 0 \right] = - \frac{2I_m}{\omega} \left[ -1 - 1 \right] \]

This gives \( I_m = \frac{2I_m}{\pi} \)

Similarly, it can be shown that \( V_m = \frac{2V_m}{\pi} \)

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**Phasor diagrams**

These are the vector diagrams in which the magnitudes represent the peak values and directions represent the phase differences between voltages and currents in the a.c circuits.

**A.C Circuits**

Fundamental components in a.c circuits.

1. Resistor
   \[ V = IR \]

2. Inductor
   \[ E = -L \frac{di}{dt} \]

3. Capacitor
   \[ Q = CV \]

**1. Circuit containing only resistor**

Circuit diagram:

Let \( V = V_m \sin \omega t \)

\[ \text{Current (I)} = \frac{V}{R} = \frac{(V_m \sin \omega t)}{R} \]

i.e., \( I = I_m \sin \omega t \)

Where \( I_m = \frac{V_m}{R} \)

\[ R = \frac{V_m}{I_m} \]

Hence the phase difference between voltage and current is, \( \phi = 0 \).

So the phasor diagram can be as shown

\[ \text{V} \rightarrow \text{I} \]

The graph showing the variation of current and voltage with time can be represented as:

**Power associated with the circuit:**

\[ P_R = \int_0^T \frac{V^2}{R} \, dt = \frac{1}{2} \int_0^T V_m \sin \omega t \cdot I_m \sin \omega t \, dt = \frac{V_m I_m}{2} \int_0^T \sin^2 \omega t \cdot dt \]

\[ = \frac{V_m I_m}{2} \cdot \left[ \frac{T}{2} - \frac{T}{2} \cos 2\omega t \right] \]

But \( \int_0^T \cos 2\omega t \, dt = 0 \)

Thus we get, \( P_R = \frac{(V_m I_m)}{2} \)

Hence \( P_R = V_{rms} \cdot I_{rms} \)

**2. Circuit containing only inductor**

Circuit diagram:

Let \( V = V_m \sin \omega t \)

The induced voltage in the inductor is \( V_{ind} = -L \left( \frac{di}{dt} \right) \)

By loop rule, \( V_m \sin \omega t - L \left( \frac{di}{dt} \right) = 0 \)

i.e., \( V_m \sin \omega t = L \left( \frac{di}{dt} \right) \)

this gives, \( \frac{di}{dt} = \frac{(V_m/L)}{\sin \omega t} \)

On integrating the above equation we get, \( \text{Current (I)} = \frac{V_m}{L} \left( \frac{\cos \omega t}{\omega} \right) \)

\[ I = \frac{V_m}{L} \cos \omega t \]

Hence \( I = I_m \sin \left( \omega t - \frac{\pi}{2} \right) \)

where \( I_m = \left( \frac{V_m}{\omega L} \right) \)

Here ‘\( \omega L \)’ represents the non-resistive opposition offered by the inductor to the flow of current in an ac circuit. It is called inductive reactance \( (X_L) \)

i.e., \( X_L = \omega L \)

Hence the phase difference between voltage and current is, \( \phi = \frac{\pi}{2} \).
Here the current lags behind the voltage by $\pi/2$.

So the phasor diagram can be as shown:

The graph showing the variation of current and voltage with time can be represented as:

**Power associated with the circuit:**

$$P_L = \frac{1}{T} \int_0^T V \sin \alpha t \cdot I_m (\sin \alpha t - \pi/2) \, dt = \frac{V \sin \alpha t}{T} \int_0^T \sin \alpha t \cdot \cos \alpha t \, dt$$

$$= -\frac{V \sin \alpha t}{2T} \left( \frac{\sin 2\alpha t}{2} \right) \, dt = -\frac{V \sin \alpha t}{2T} \cdot 0 \text{ as } \int_0^T \sin 2\alpha t \, dt = 0$$

Thus we get, $P_L = 0$ i.e., an ideal inductor does not consume any power.

(Note: this is why they are used as ‘choke coils’ in ac circuits to reduce current without power loss, in the place of resistors)

3. Circuit containing only capacitor

**Circuit diagram:**

Let $V = V_m \sin \omega t$

We have $Q = CV$

Differentiating we get, $\frac{dQ}{dt} = C \frac{dV}{dt}$

i.e. $I = C V_m \cos \omega t \cdot \alpha$

$I = \frac{V_m}{\sqrt{C \omega}} \sin (\omega t + \pi/2)$

Hence $I = I_m \sin (\omega t + \pi/2)$ where $I_m = \frac{V_m}{\sqrt{C \omega}}$

Thus $X_C = \frac{1}{C \omega}$ gives the non-resistive opposition to the flow of current called capacitive reactance.

Hence the phase difference between voltage and current is, $\phi = \pi/2$.

Here the current leads the voltage by $\pi/2$. So the phasor diagram

Is shown beside:-

The graph showing the variation of current and voltage with time can be represented as:

**Power associated with the circuit:**

$$P_L = \frac{1}{T} \int_0^T V \sin \alpha t \cdot I_m (\sin \alpha t + \pi/2) \, dt = \frac{V \sin \alpha t}{T} \int_0^T \sin \alpha t \cdot \cos \alpha t \, dt$$

$$= \frac{V \sin \alpha t}{2T} \left( \frac{\sin 2\alpha t}{2} \right) \, dt = \frac{V \sin \alpha t}{2T} \cdot 0 \text{ as } \int_0^T \sin 2\alpha t \, dt = 0$$

Thus we get, $P_L = 0$ i.e., an ideal capacitor does not consume any power.

3. L-C-R series circuit

**Circuit diagram:**

Let $V = V_m \sin \omega t$

Since the components are in series, the current, $I$ is the same throughout. Let $V_L$, $V_C$, and $V_R$ be the p.d’s across the resistor, inductor and capacitor respectively. Assuming $V_L$ to be greater than $V_C$, the phasor diagram can be drawn as,

The resultant of the above vector diagram is:
By parallelogram law, \[ V = V_L - V_C \]

The resultant voltage is given by, \[ V = \sqrt{V_L^2 + (V_C - V_L)^2} \]

But \[ V_R = IR, \quad V_I = IX_L \quad \text{and} \quad V_C = IX_C \]

Thus \[ V = IR \sqrt{R^2 + (X_C - X_L)^2} \]

Hence \[ \frac{V}{I} = \sqrt{R^2 + (X_C - X_L)^2} \]

This gives the total opposition to the flow of current in an a.c. circuit, including resistance and reactances. It is called the impedance (Z) of the circuit.

Thus the impedance is given by \[ Z = \sqrt{R^2 + (X_C - X_L)^2} \]

Also the phase difference between the voltage and current is given by \[ \tan\phi = \frac{V_L - V_C}{V_R} \]

Hence, current in the circuit can be expressed as \[ I = I_m \sin(\omega t - \phi) \]

The negative sign indicates that the inductive reactance dominates capacitive. i.e. current lags behind voltage.

**Resonance:** It is the condition at which the inductive and capacitive reactances are equal so that the current in the circuit is maximum.

We know \[ I_n = \frac{V}{Z} \]

Current is maximum, when impedance is minimum.

But \[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

So ‘Z’ is minimum when \[ X_L = X_C \]

Hence \[ Z_{min} = R \]

\[ \omega L = 1/\omega C \quad \text{or} \quad \omega^2 = 1/\omega L \]

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**Additional notes:**

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Thus \((I_m)_{\text{max}} = \frac{V_m}{R}\)

Resonant frequency \(\omega_r = \frac{1}{\sqrt{LC}}\) and \(V_r = \frac{1}{2\pi \sqrt{LC}}\)

Frequency response curves with current
The variation of peak current with different frequencies can be represented for different values of resistances as shown beside:

\[\text{VARIATION OF RESISTANCE, REACTANCE AND IMPEDANCE WITH FREQUENCY}\]

Resistance is independent of the frequency.

\(X_c = \frac{1}{\omega C}\) \(\text{So, } X_c \alpha \frac{1}{\omega}\)

\(X_L = \omega L\) \(X_L \alpha \omega\)

\[\text{POWER AT RESONANCE}\]

We have Power \(P = V_{\text{rms}} \cdot I_{\text{rms}} \cdot \frac{R}{2}\)

But \(V_{\text{rms}} = \frac{V_m}{\sqrt{2}}\), \(I_{\text{rms}} = \frac{I_m}{\sqrt{2}}\) and \(I_m = \frac{V_m}{Z}\)

So \(P = \frac{V_m^2}{2Z} R\) \(\text{------------ (1)}\)

At resonance, \(Z\) is minimum. Thus power also is maximum at resonance.

Hence \(P_{\text{max}} = \frac{V_m^2}{2Z} R\)

Eqn(1) becomes \(P = P_{\text{max}} \frac{R^2}{2Z}\)

The frequency response curve with power can be drawn as:

The frequencies at which the power in the circuit is half of Its maximum value are called 'half power points'.

The difference between half power points is called Bandwidth \((2\Delta \omega)\)

\[2\Delta \omega = \omega_2 - \omega_1\]

[For the derivation of the expression for band width, \(2\Delta \omega = \frac{R}{L}\), refer the note book]

\[\text{Q-FACTOR (Quality factor)}\]

It measures the sharpness of the resonance curve. It is defined as the ratio of the resonant frequency to the bandwidth.

\[Q = \frac{\omega_r}{2\Delta \omega}\]

Using the above equations we get,

\[Q = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} \quad \text{so, } Q = \frac{1}{R \sqrt{LC}}\]
Radio Tuner

Tuning is the process of receiving the frequency of a desired radio station. It consists of an inductor and a variable capacitor. During tuning, the value of capacitance is changed. At a given position the value of capacitance is such that it satisfies for a given frequency in the expression for resonant frequency, \[ \nu_r = \frac{1}{2\pi \sqrt{LC}}. \]

Since the Q-factor of the circuit is adjusted to be very high, the maximum power reception occurs only at this frequency. Thus tuning is achieved.

**L-C OSCILLATIONS**

A capacitor, initially charged to \( Q_0 \), and is disconnected from the battery. Further it is connected across an inductor. This causes to and fro motion of electrons in the circuit, as illustrated below:

This gives electrical oscillations of frequency \( \nu = 1/T \).

**Mathematical Treatment**

\[ \text{[Qn. Show that electric charges execute S.H.M in an L-C circuit. Hence derive an expression for the frequency of oscillations. Show that the total energy in the circuit is conserved.]} \]

\[ \text{----- Refer Note book--------------} \]

**Advantages of A.C**

(i) Cost of production is low.
(ii) It can be transmitted to distant places with minimum power loss, with the help of transformers.
(iii) It can easily be converted into d.c, wherever needed.
(iv) Current in the circuits can be controlled with the help of choke coils.

**Disadvantages of A.C**

(i) It is more dangerous.
(ii) It cannot be used in the processes like electroplating, making permanent magnets, etc.
(iii) Energy loss can occur in the case of high frequency a.c due to **skin effect**. [ It is the effect in which high frequency a.c confines through the surface of a conductor, when transmitted.]

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